A Void-Fraction Correlation for Vertical and Horizontal Bulk-Boiling of Water

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The knowledge of void fraction, the relative volumes of gas and liquid phases at a given location in a tube, is important for the calculation of the mean density of a two-phase fluid, which is needed for the estimation of hydrostatic, acceleration, and friction pressure drops in boiling loops. For this reason an adequate correlation is necessary for reboiler and evaporator design in chemical process plants and for steam generator design. The prediction of mean density of a two-phase flow is also necessary for estimating the reactivity of boiling water nuclear reactors.

Many empirical as well as theoretical models have been proposed during the last quarter century; but most of these models are based on rather fundamental simplifying assumptions, frequently with respect to the flow regime, and therefore the results cannot be of general applicability to boiling phenomena where several different flow regimes are usually traversed in a single tube, from bubbly to separate flow.

Void fraction is also closely related to both the frictional and accelerational pressure drops so that precise evaluation of void fraction could be helpful in elucidation of this relationship.

A ONE-DIMENSIONAL TWO-PHASE FLOW MODEL

The basic range of applicability of the proposed model is to the bulk-boiling regime, and it applies to a range of flow regimes from detached voidage through churn to separate flow.

For diabatic experiments, the quality x is the thermodynamic flow quality defined by means of the equation

$$x = (i - i_2)/(i_1 - i_2) \tag{1}$$

where i without subscript is the enthalpy of the two-phase flow leaving a given control volume. During a diabatic experiment this will equal the enthalpy of the flow entering a control volume plus the thermal energy added to the control volume.

For an adiabatic experiment the quality is defined as the fractional mass flow of vapor out of the control volume. If vapor and liquid are in thermodynamic equilibrium, these two definitions are equivalent.

The flow is treated as if it is one-dimensional and average velocities are used to express both the velocity of the liquid and of the vapor phase. The one-dimensional, two-phase continuity equation is

$$\alpha = \frac{1}{1 + \frac{1-x}{x}\left(\frac{\rho_2}{\rho_1}\right)^{1/n}} \tag{2}$$

where α is the void fraction, x the quality, ρ_1/ρ_2 the density ratio, and n the velocity ratio v_2/v_1. Equation (2) is derived from basic definitions by Wallis (1969). These definitions are the same as given above, except for the thermodynamic quality. The reason for introducing the thermodynamic quality to supplement the usual flow quality is that it is this quality which is usually observed in diabatic experiments, and it is this quality which is correlated with void fraction. Since the density ratio ρ_2/ρ_1 is determined by the thermodynamic state, it is possible to calculate velocity ratio n from the observed void fraction at a given quality using Equation (2). The flow regime need not be specified since no assumption was made about it in deriving the equation. Of course the value of n is dependent on the flow regime. Assume that the velocity ratio may be expressed as a power function of ρ_2/ρ_1

$$n = (\rho_2/\rho_1)^{-L} \tag{3}$$

where L is an empirical parameter to be determined from experiments, where the value of L may vary from $L \to -\infty$ where sessile bubbles on the tube make $n = 0$, to an upper limit of 0.50 as found from experimental data. For the present, the question of the value of L may be left open and the expression for n in Equation (3) may be substituted in Equation (2). Therefore

$$\alpha = \frac{1}{1 + \frac{1-x}{x}\left(\frac{\rho_1}{\rho_2}\right)^{L-1}} \tag{4}$$

now L may be calculated from existing void-fraction data and plotted as a function of log $[x/(1 - x)]$. Figure 1 shows such a plot for the data of Rouhani and Becker at $p = 3104\ \text{kN/m}^2$ (1963). The set of data chosen supports a straight-line fit unequivocally. Other data justify a straight line less well, but adequately. All data indicate that the lines pass through the point (0.50,0); this corresponds to the value for the critical two-phase mass velocity as shown by Fauske (1961). The intercept on the log $[x/(1 - x)]$ axis, when $L = 0$, and the flow appear to be homogeneous is designated by the subscript 0: that is, the intercept is symbolized as $\log[x_0/(1 - x_0)]$. Therefore, the straight line on Figure 1 may be represented by the equation

$$L = \frac{1}{2}\left[1 - \frac{\log x/(1-x)}{\log x_0/(1-x_0)}\right] \tag{5}$$

even the data where $x = x_0$, and $L = 0$ fit this equation. When $x = x_0$ and $L = 0$, Equation (4) yields

$$x_0/(1 - x_0) = \alpha_0/(1 - \alpha_0) \cdot (\rho_2/\rho_1) \tag{6}$$

if this value for $x_0/(1 - x_0)$ is substituted in Equation (5), the result is

$$L = \frac{1}{2} - \frac{\frac{1}{2}\log x/(1-x)}{\log \alpha_0/(1-\alpha_0) - \log(\rho_2/\rho_1)} \tag{7}$$

substituting this expression for L in Equation (4), rearranging, changing from exponential to log function, we get after some mathematical manipulation

$$\alpha = \frac{1}{1 + \left(\frac{\rho_1}{\rho_2}\right)^{1/2} \left(\frac{1-x}{x}\right)^m}$$

where

$$m = \frac{\tfrac{1}{2}\log(\rho_1/\rho_2) - \log[\alpha_0/(1-\alpha_0)]}{\log(\rho_1/\rho_2) - \log[\alpha_0/(1-\alpha_0)]} \qquad (8)$$

it is clear from the latter expression that when ρ_1/ρ_2 becomes very large compared to $\alpha_0/(1 - \alpha_0)$, at low pressures, then m approaches ½ as a limit; and when ρ_1/ρ_2 approaches the critical point, then m approaches one, and the void fraction becomes equal to the quality. Generally m will have a value lying between these limits, or $\tfrac{1}{2} < m < 1$.

Using the data compiled by Smith (1970), with the assumption that the random errors in the measurements of void-fractions calculated from the functional relationship, Equation (8); a best value was found for each pressure. Equation (8) was programmed on digital computer, IBM 360/50, and the sum of the squares of the deviations from experimental data was computed for various values of α_0 until a minimum was attained.

Since the result did not warrant a term allowing for variation in α_0 with pressure, a least square computation of the α_0 which best fits all the data was done. A value of $\alpha_0 = 0.302$ with a variance of 178×10^{-5} was computed.*

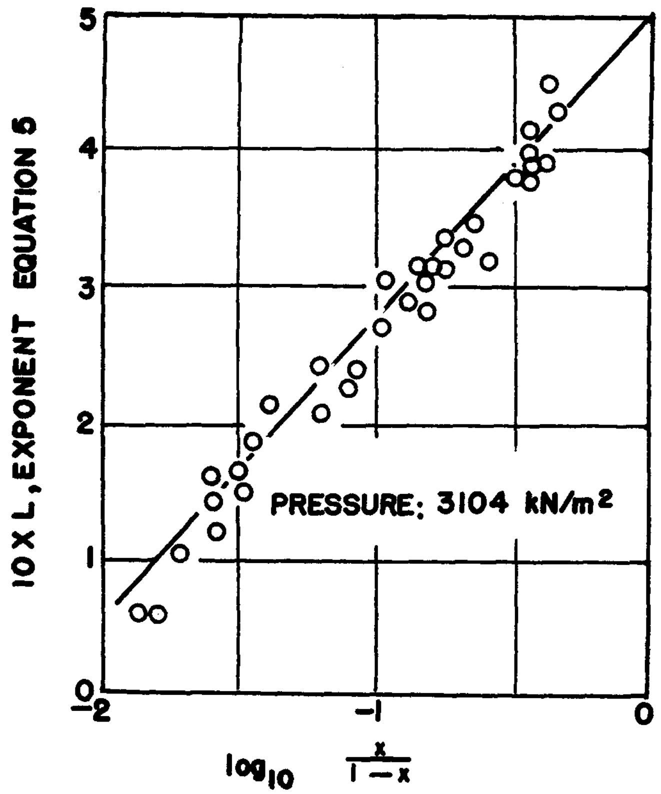

Fig. 1. Exponent n vs. $\log_{10} x/(1 - x)$.

* Supplementary material has been deposited as Document No. 02562 with the National Auxiliary Publications Service (NAPS), c/o Microfiche Publications, 440 Park Ave. So., N.Y., N.Y. 10016 and may be obtained for $2.00 for microfiche or $11.80 for photocopies.

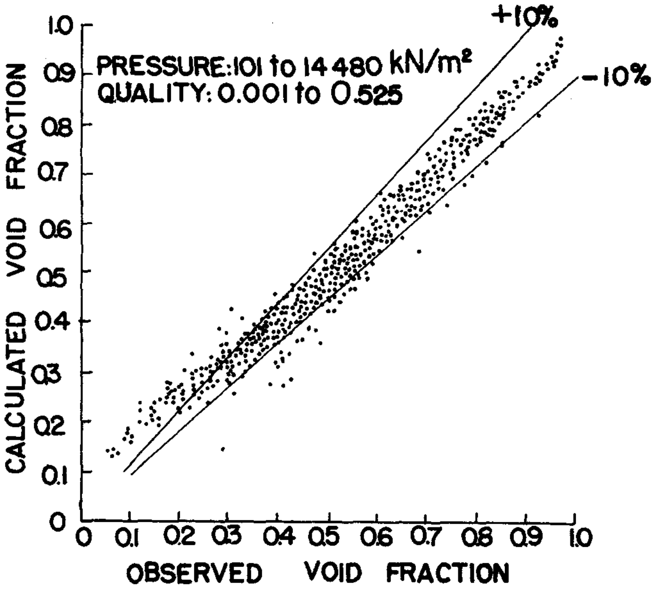

Fig. 2. Void fraction: calculated vs. observed values ($\alpha_0 = 0.302$).

Figure 2 shows a plot of void fractions calculated with the value of α_0 above, against observed void fractions. The fit is a distinct improvement on the analogous plot presented by Smith (1970).

CONCLUSION

Equation (8), with $\alpha_0 = 0.302$ represents the void-fraction data for water in equilibrium contact with air or steam in concurrent flow within circular tubes, diabatic or adiabatic flow, qualities from 0.001 to 0.5, and pressures from 101 to 14480 kN/m².

NOTATION

i = enthalpy, kJ/kg
L = exponent in Equation (3), dimensionless
m = exponent in Equation (8), dimensionless
n = velocity ratio, v_2/v_1, dimensionless
v = velocity (average phase velocity), m/s
x = quality, dimensionless

Greek Letters

α = void fraction, dimensionless
ρ = density, kg/m³

Subscripts

0 = when $v_1 = v_2$
1 = liquid phase
2 = vapor phase

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